Time: 3 Hours] **[Max. Marks: 80**

Note: 1) All Questions are compulsory

2) Figures to the right indicate full marks

Q.1 (a) Attempt the following **(10)**

(i) Define Topology, Basis for a Topology.
Let X be a set; let $\mathcal{B}$ be a basis for a topology $\mathcal{T}$ on X. Then show that $\mathcal{T}$ equals the collection of all unions of elements of $\mathcal{B}$.

(b) Attempt any two **(10)**

(i) Define Open set, Interior of a set.
If A is an open set, then show that $Int\ A = A$, where $Int\ A$ is interior of A.

(ii) Define discrete topology on a non-empty set X.
Show that every subset U of X (with discrete topology) is open in X. Show that this implies that every subset V of X is closed in X.

(iii) Show that if U is open in X and A is closed in X, then $U - A$ is open in X, and $A - U$ is closed in X.

Q.2 (a) Attempt the following **(10)**

(i) Define Connected Space.
Let A be connected subspace of X. If $A \subset B \subset \bar{A}$, then show that B is a connected subspace of X.

(b) Attempt any two **(10)**

(i) Prove that the union of connected subspaces of X that have a point in common is connected. If no point is common (to all the connected subspaces), then show by an example that the union may not be connected.

(ii) When do we say that a space X is first countable? Can we say that $\mathbb{R}$ with the standard topology is first countable? Justify.

(iii) State and prove Intermediate Value Theorem.

Q.3 (a) Attempt the following **(10)**

(i) Define compact set of a topological space. Let $A_1, \dots, A_n$ be compact subsets of a topological space (X, τ). Prove that $\bigcup_{i=1}^{n} A_i$ is compact. Is $\bigcap_{i=1}^{n} A_i$ also compact? Justify your answer.

(b) Attempt any two **(10)**

(i) Let $f: X \to Y$ be a bijective continuous function. If X is compact and Y is Hausdorff, then prove that f is a homeomorphism.

(ii) Define $\tau_f = \{A \subseteq X: A = \emptyset, X - A \text{ is finite}\}$, the cofinite topology on a nonempty set X. Prove that every subset of X is compact.

(iii) Let Y be a subspace of X. If every covering of Y by open sets in X contains a finite subcollection covering Y, then prove that Y is compact.

Q.4 (a) Attempt the following **(10)**

(i) Define regular and normal spaces. Suppose one-point sets are closed in X. Prove that X is regular, if X is normal. Further, show that a regular space need not be normal.

(b) Attempt any two **(10)**

(i) State Urysohn Lemma. Give a direct proof of Urysohn's lemma for metric spaces.

(ii) Prove that homeomorphic image of a normal space is normal.

(iii) Let X be a connected normal space such that one-point sets be closed in X. If X contains more than one point, prove that X is uncountable.
